Theoretical Predictions of Giant Resonances in ^{94}Mo

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Introduction

Giant resonances are collective excitations of nuclei between 10 and 40 MeV. They are classified by their multipolarity L, Spin S, and Isospin T. In this work, we examine the Isoscalar (T=0) Giant Resonances in ^{94}Mo of L=0, 1, 2, and 3. The strength functions of these resonances are calculated within Hartree-Fock based Random Phase Approximation (HF RPA) using thirty-three common Skyrme interactions. The centroid energies are plotted against Nuclear Matter (NM) properties in hopes of constraining them.

Nuclear Equation of State

The nuclear equation of state gives the binding energy per nucleon $E[\rho_p,\rho_n]$ as a function of the proton density ρ_p and neutron density ρ_n. Most nuclei we study are close to symmetric, so we split up our functional based on symmetry.

$$E[\rho_p,\rho_n]=E_0[\rho]+E_{sym}[\rho]\left(\frac{\rho_n-\rho_p}{\rho}\right)^2, \qquad \rho=\rho_p+\rho_n,$$

where $E_0[\rho]$ is the energy per nucleon of symmetric nuclear matter and $E_{sym}[\rho]$ is the symmetry energy. We know experimentally that $E_0[\rho]$ has a minimum at $\rho_0 \approx 0.16\text{fm}^{-3}$ with an energy per nucleon of about 16MeV. We can expand both of these functions as Taylor series of nuclear density around ρ_0. We name the expansion coefficients as properties of NM [1].

$$E_0[\rho]\approx E[\rho_0]+\frac{1}{18}K\left(\frac{\rho-\rho_0}{\rho_0}\right)^2, \qquad E_{sym}[\rho]\approx J+\frac{1}{3}L\left(\frac{\rho_n-\rho_p}{\rho_0}\right)+\frac{1}{18}K_{sym}\left(\frac{\rho_n-\rho_p}{\rho_0}\right)^2,$$

$$K=9\rho^2\frac{d^2E_0[\rho]}{d\rho^2}\bigg|_{\rho_0}, \quad J=E_{sym}[\rho_0], \quad L=3\rho\frac{dE_{sym}[\rho]}{d\rho}\bigg|_{\rho_0}, \quad K_{sym}=9\rho^2\frac{d^2E_{sym}[\rho]}{d\rho^2}\bigg|_{\rho_0}.$$

The Isoscalar Giant Monopole Resonance (L=0) (ISGMR) is sensitive to K, the incompressibility coefficient of nuclear matter. The ISGMR is a breathing mode that acts similarly to a simple harmonic oscillator. It is possible that, with computation, we may find that other resonances are sensitive to other NM properties.

Hartree-Fock Method

The Hartree-Fock method assumes that we can treat all of the particles as moving independently in a central potential. The total nuclear wave function can then be written as a Slater determinant of single particle wave functions $\phi_i(\vec{x})$ [4].

$$\Psi(\vec{x}_1,\vec{x}_2,\ldots,\vec{x}_n)=\frac{1}{\sqrt{n!}}\begin{vmatrix}\phi_1(\vec{x}_1) & \cdots & \phi_1(\vec{x}_n)\\ \vdots & \ddots & \vdots\\ \phi_n(\vec{x}_1) & \cdots & \phi_n(\vec{x}_n)\end{vmatrix}$$

The energy expectation value $\langle\Psi|\hat{H}|\Psi\rangle$ is then minimized for a general Hamiltonian of the form $\hat{H}=\sum_{i=1}\frac{p_i^2}{2m}+\sum_{i<j}\hat{V}_{ij}$, where V_{ij} is the two-body interaction. To minimize this expectation value, we vary the single particle states. Varying the wave functions and setting the varied expectation value to zero gives the Hartree-Fock equations.

$$\phi_i \to \phi_i+\delta\phi_i,$$

$$-\frac{\hbar^2}{2m}\nabla^2\phi_i(\mathbf{r})+\phi_i(\mathbf{r})U_H(\mathbf{r})-\int d^3\mathbf{r}'\,U_F(\mathbf{r},\mathbf{r}')\phi_i(\mathbf{r}')=e_i\phi_i(\mathbf{r}),$$

$$U_H(\mathbf{r})=\sum_{j=1}^{A}\int d^3\mathbf{r}'\,\phi_j^*(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\phi_j(\mathbf{r}'), \qquad U_F(\mathbf{r},\mathbf{r}')=\sum_{j=1}^{A}\phi_j^*(\mathbf{r}')V(\mathbf{r},\mathbf{r}')\phi_j(\mathbf{r}).$$

Here e_i are the single particle energies. These equations are solved iteratively to obtain energies, the central potential, and the single particle wave functions.

There are many different interactions that can be used to model the nuclear force. Skyrme interactions are momentum dependent contact interactions with ten parameters x_i, t_i, α, and W_0 that are determined by a fit to experimental data.

$$V_{ij}=t_0(1+x_0P_{ij}^\sigma)\delta(\mathbf{r}_i-\mathbf{r}_j)+\frac{1}{2}t_1(1+x_1P_{ij}^\sigma)\left[\vec{k}_{ij}'^2\delta(\mathbf{r}_i-\mathbf{r}_j)+\delta(\mathbf{r}_i-\mathbf{r}_j)\vec{k}_{ij}^2\right]+$$
$$t_2(1+x_2P_{ij}^\sigma)\vec{k}_{ij}'\delta(\mathbf{r}_i-\mathbf{r}_j)\vec{k}_{ij}+\frac{1}{6}t_3(1+x_3P_{ij}^\sigma)\rho^\alpha\left(\frac{\mathbf{r}_i+\mathbf{r}_j}{2}\right)\delta(\mathbf{r}_i-\mathbf{r}_j)+iW_0\vec{k}_{ij}'\left(\vec{\sigma}_i+\vec{\sigma}_j\right)\delta(\mathbf{r}_i-\mathbf{r}_j)\vec{k}_{ij}$$

There are hundreds of published sets of values for these parameters and each one gives new values of NM properties. We will plot our strength functions for the KDE0 interaction in particular, within the Hartree-Fock (HF) based Random Phase Approximation (RPA).

Strength Functions

All of our data comes from scattering experiments. A scattering operator F_L with radial and angular dependence maps the ground state to excited states. We define the strength function S(E) below. The sum over j sums over all RPA states, and $|0\rangle$ is the HF RPA ground state,

$$S(E)=\sum_j\left|\langle 0|F_L|j\rangle\right|^2\delta(E_j-E_0).$$

The scattering operator for isoscalar resonances is written below. The form of f(r) for the different resonances can be found in [1],

$$F_L=\sum_i f(r_i)Y_{L0}(i).$$

We can take energy moments m_k of this function and define the centroid energy, E_{cen},

$$m_k=\int_0^\infty dE\,E^kS(E), \qquad E_{cen}=\frac{m_1}{m_0}.$$

In figure 1, the experimental strength functions [2] are plotted with a Gaussian fit (red) along with the calculated KDE0 strength functions (purple). Each one corresponds to a resonance of different L: E0 represents L0, E1 represents L1, and so on. In order to isolate the Isoscalar Giant Resonances, the limits of the m_k integral are chosen carefully. They are 9-40 MeV for the ISGMR and Isoscalar Giant Quadrupole Resonance (ISGQR), 20-36 MeV for the high part of Isoscalar Giant Dipole Resonance (ISGDR), and 14-40 MeV for the High Energy Octopole Resonance (HEOR).

In figure 2, we plot the centroid energies of the multipoles in ^{94}Mo against K, m*, and J from thirty-three Skyrme interactions found in [3]. The dotted lines show experimental uncertainties [2]. The ISGMR has a correspondence with K, but it this correspondence weakens as L increases. In the opposite way, m* correspondence becomes stronger for higher L. The symmetry energy J has no obvious correspondence with any of these resonances.

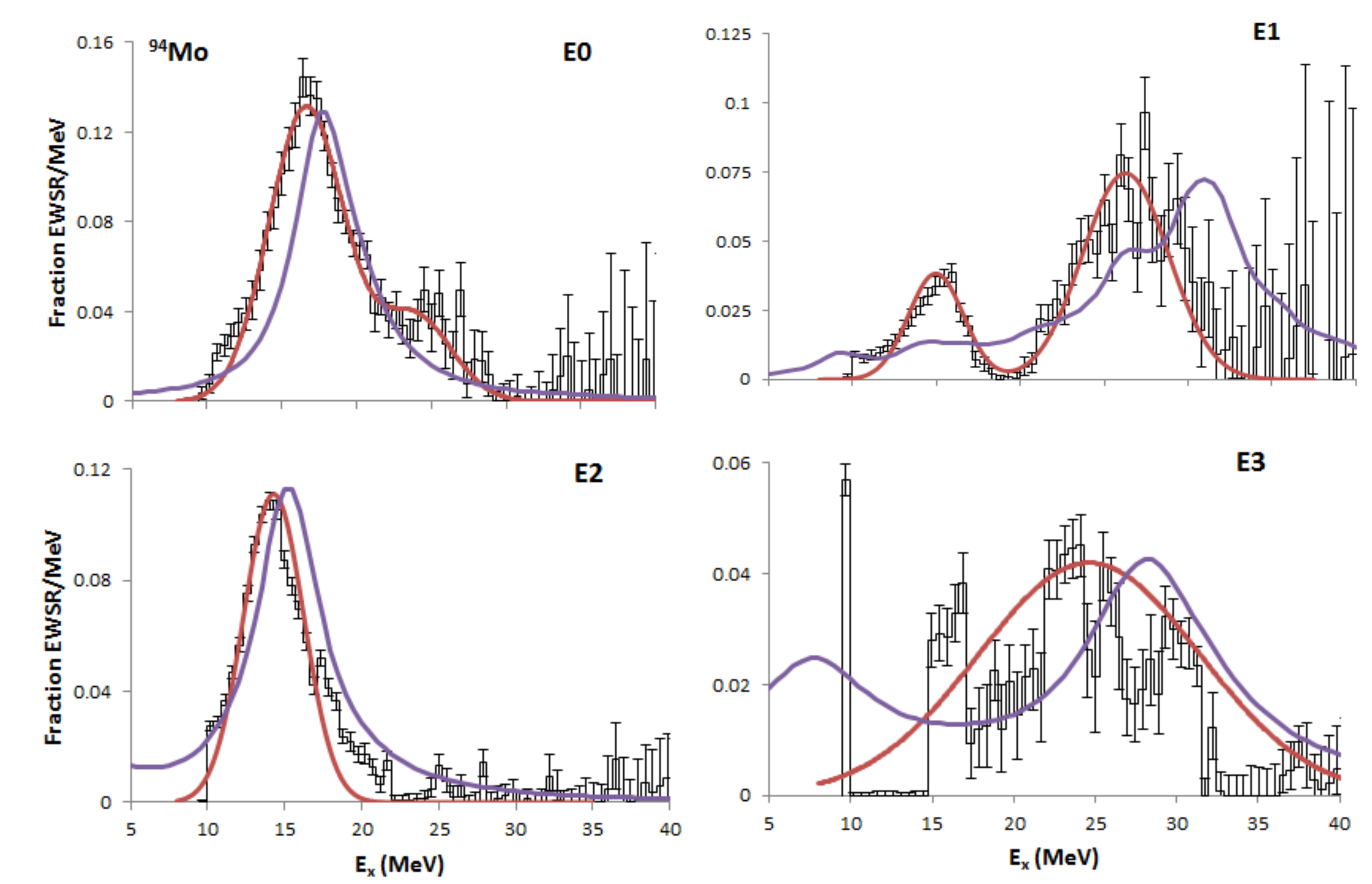

Figure 1. Experimental strength functions of each multipolarity in ^{94}Mo are given a Gaussian fit (red) and plotted with the KDE0 strength function (purple). Data is taken from [2].

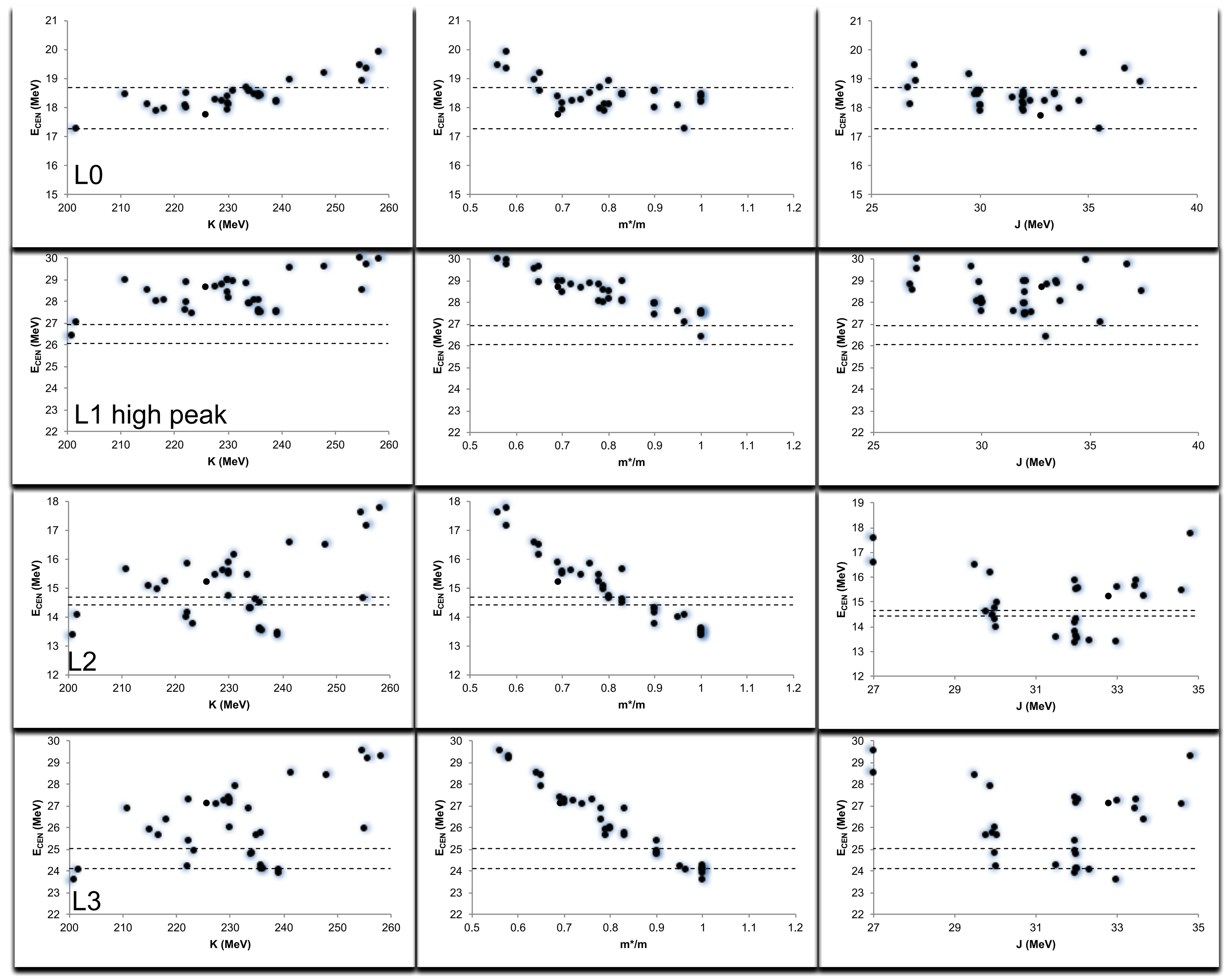

Figure 2. Centroid energies for different multipolarities L in ^{94}Mo are plotted against K, m*/m, and J. The dotted lines show experimental uncertainties [2].

Conclusions

Our calculations are consistent with the currently accepted value of K = 240 ± 20 MeV. There also exists a preference for m*/m = 0.8 ± 0.1. No other nuclear matter properties could be determined with a reasonable uncertainty from this data.

The strength function of the monopole has a high energy peak that is not reproduced by the calculations.

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